Impact of Measurement Errors on the Closed Loop Power Control for CDMA Systems

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Abstract—In this paper, we present a simple analytical model to evaluate the impact of measurement errors on the closed-loop power control for code division multiple access (CDMA) systems. By introducing a new performance measurement, the probability of false command in power control, $P_{FC}$, a closed-form formula for calculating $P_{FC}$ with consideration of measurement errors is presented. Furthermore, we derive a bit error rate (BER) performance bound in terms of $P_{FC}$ for the CDMA system with the closed-loop power control (CLPC). The proposed analytical approach can quantitatively evaluate the performance of the CLPC taking into account of the effects of measurement errors and Doppler frequency under a Rayleigh fading channel. Through simulation and analysis, we show that the proposed analytical BER bound can accurately estimate the BER performance of the CLPC under the impact of measurement errors. Interestingly, we find that the CLPC is less sensitive to measurement errors due to the non-linear operation in the one step up/down power control scheme compared with the variable-step size power control.

Index Terms—CDMA System, Power Control, Measurement Error.

I. INTRODUCTION

Accurate power control is one of the key technologies to achieve high capacity code division multiple access (CDMA) systems. Power control errors may be resulted from many factors [1], such as loop delay [2], quantization errors [3], multi-path fading [4], [5], link-quality measurement errors [6], and feedback errors [7]. Although the closed-loop power control (CLPC) in the CDMA system has been studied extensively in the literature [3], [6], [8], [9], [10], fewer papers have analyzed the performance of power control scheme subject to signal-to-interference ratio (SIR) measurement errors.

Previous works about the impact of measurement errors on power control can be summarized as follows. The impact of measurement errors on the open-loop power control was studied in [6]. The authors in [1] discussed the filtering effect in the measurement scheme for the WCDMA systems. In [10], [11], [12], the issue of joint minimization of SIR measurement errors and power control errors are investigated in the form of a stochastic control problem, but the SIR measurement errors in [10], [11], [12] are modeled as white Gaussian noise. However, in [13], it has been found that measurement errors tend to be log-normal distributed in cellular channel with Rayleigh fading and shadowing.

The goal of this paper is to develop a simple and accurate analytical model to evaluate the impact of log-normal distributed SIR measurement errors on the CLPC of CDMA systems. The contributions of this work can be summarized in two folds. First, to evaluate the impact of measurement errors on the closed-loop power control, we introduce a new performance measurement called the probability of false power control command, $P_{FC}$. The motivation of introducing the new parameter, $P_{FC}$, is because the feedback power control command is the key to the accuracy of power control scheme. We will present a close-form formula for calculating $P_{FC}$ in terms of measurement errors. Second, we propose a simple BER bound in terms of $P_{FC}$. Hence, by using the proposed analytical approach, the performance for different SIR measurement schemes on the closed-loop power control in the CDMA system can be easily obtained.

The rest of this paper is organized as follows. In Section II, we briefly introduce power control and define the probability $P_{FC}$. Section III derives the closed-form formula of $P_{FC}$ in terms of measurement errors. In Section IV, we derive a simple BER bound for the CLPC in CDMA systems with the parameter $P_{FC}$. Section V shows some analytical and simulation results. Section VI gives concluding remarks.

II. PROBABILITY OF FALSE POWER CONTROL COMMAND

A. Background

Generally speaking, we can categorize power control models into two kinds. The first one is the open-loop power control and the other one is the closed-loop power control. In
the former scheme, a mobile terminal decides its own trans-
mission power by comparing the desired received SIR and 
the target SIR to compensate path loss and shadowing. On 
the other hand, the CLPC makes the base station determine 
down the power control command by comparing the re-
ceived SIR with the target SIR. The block diagram of the 
CLPC scheme is shown in Fig.1. As shown in this figure, 
the CLPC has two feedback loops where the inner loop is 
for fast power adjustment, while the outer loop is for setting 
the target $E_b/N_0$.

**B. Probability of False Power Control Command**

Because the accuracy of power control commands will 
strongly influence the performance of the closed loop power 
control, it is important to investigate the impact of measure-
ment errors on the false power control command. Figure 2 
illustrates how measurement errors will influence the power 
control command. For example, as shown in Fig. 2, if the 
actual SIR value is below the target SIR value, the CLPC 
will issue a “power up” command to increase the trans-
mision power. For a small measurement error, the power con-
trol command will still be correct since the measured SIR is 
below the target SIR as shown in the left hand side of the 
figure. However, if a large measurement error causes the re-
ceived SIR to become larger than the target SIR, then power 
control command may change to be a wrong “power down” 
command, as shown in the right hand side of the figure.

In order to evaluate the effect of measurement errors on 
the closed-loop power control scheme, we define the proba-
bility of false command ($P_{FC}$) in power control as follows.

$$P_{FC} = \text{Prob}\{ \text{sgn} (SIR_T - SIR_M) \neq \text{sgn} (SIR_T - SIR_A) \}$$  \hspace{1cm} (1)

where $\text{sgn}(x)$ is the operator to choose the sign of $x$, $SIR_M$ 
is the measured SIR, $SIR_A$ is the actual SIR value, and the 
$SIR_T$ is the target SIR. From this definition, one can see 
that the measurement error makes the sign of (1) change, 
thereby issuing a false command in power control.

**III. ANALYSIS**

In this section, we will derive an analytical formula to 
evaluate the probability of false command in power control. 
To begin with, we model the power control error (PCE) and 
measurement errors as independent log-normal distributed 
random variable as [9], [13]. Denote the SIR measurement 
error by a random variable, $Y$, and the power control error by 
a random variable, $X$. In the following, we will consider all 
the variables in the dB domain. Denote $SIR_T$ and $SIR_A$ as 
the target SIR and the actual SIR, respectively. Then $SIR_A$
can be expressed as the summation of $SIR_T$ and power con-
trol error, $X$, i.e.

$$SIR_A = SIR_T + X.$$  \hspace{1cm} (2)

The measured SIR, denoted as $SIR_M$, can be expressed as 
the summation of $SIR_A$ and the measurement error, $Y$, i.e,

$$SIR_M = SIR_A + Y.$$  \hspace{1cm} (3)

From (2) and (3), we know

$$SIR_M = SIR_T + X + Y.$$  \hspace{1cm} (4)

Substituting (2) and (4) into (1), we find that a false power 
control command occurs when the following condition is 
sustained:

$$\text{sgn}(SIR_T - (SIR_T + X + Y)) \neq \text{sgn}(SIR_T - (SIR_T + X))$$  \hspace{1cm} (5)

or

$$\text{sgn}(X + Y) \neq \text{sgn}(X).$$  \hspace{1cm} (6)

Hence, the probability of false power control command ($P_{FC}$) can be written as

$$P_{FC} = P(X + Y < 0 | X > 0) P(X > 0) + P(X + Y > 0 | X < 0) P(X < 0) = P(X + Y < 0, X > 0) + P(X + Y > 0, X < 0).$$  \hspace{1cm} (7)
Let \( x' = x/\sigma_X \) and \( y' = y/\sigma_Y \). Then we can express \( P_{FC} \) as

\[
P_{FC} = 2 \int_0^\infty \int_{-\infty}^\infty \frac{1}{2\pi} \exp\left(-\frac{x'^2}{2\sigma_X^2}\right) \cdot \exp\left(-\frac{y'^2}{2\sigma_Y^2}\right) dy' dx' .
\]

(11)

Next, we change the coordinate axis to polar coordinate axis by letting \( x' = r \cos \theta \) and \( y' = r \sin \theta \). Then \( P_{FC} \) can be written as

\[
P_{FC} = 2 \int_{\tan^{-1} \frac{\sigma_X}{\sigma_Y}}^{\pi/2} \int_0^\infty \frac{1}{2\pi} e^{-\frac{r^2}{2}} r dr d\theta
= \frac{1}{2} - \frac{\tan^{-1} \frac{\sigma_X}{\sigma_Y}}{\pi} .
\]

(12)

Now we want to analyze the impact of \( P_{FC} \) on the BER performance. From [14], we know that for the received bit energy to noise density ratio \( \gamma_b \), the bit error rate of BPSK modulated signals in Rayleigh fading channel is

\[
P_{b1} = \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma_b}{1 + \gamma_b}} \right) .
\]

(13)

In the AWGN channel, the BER of BPSK modulated signals is

\[
P_{b2} = Q(\sqrt{2\gamma_b})
\]

(14)

where

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt .
\]

(15)

Since the purpose of power control is aimed to remove Rayleigh fading, we expect that the BER performance of the CLPC will be closed to (14) if the power control perfectly removes the impact of Rayleigh fading; and on the other hand, the BER performance of the CLPC will be close to (13) if there is no power control. Consequently, we propose an upper bound on the BER performance for the closed-loop power control subject to measurement errors as follows.

\[
P_{b,FC} \leq 2P_{FC}P_{b1} + (1 - 2P_{FC})P_{b2}
\]

\[
\leq P_{FC} \times \left( 1 - \sqrt{\frac{\gamma_b}{1 + \gamma_b}} \right) + (1 - 2P_{FC})Q(\sqrt{2\gamma_b}) ,
\]

(16)

where \( P_{b1} \) and \( P_{b2} \) are defined in (13) and (14). Note that because the maximum of \( P_{FC} \) is 0.5, we use 2\( P_{FC} \) to approximate the case without power control. We will evaluate the accuracy of the proposed upper bound (16) via simulation in the next section.
TABLE I
SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spreading Factor</td>
<td>4</td>
</tr>
<tr>
<td>Doppler Frequency</td>
<td>5-30Hz</td>
</tr>
<tr>
<td>Power Control Period</td>
<td>0.667mSec</td>
</tr>
<tr>
<td>Power Control Step Size</td>
<td>1 dB</td>
</tr>
<tr>
<td>Target Eb/No</td>
<td>5 dB</td>
</tr>
<tr>
<td>Modulation Scheme</td>
<td>BPSK</td>
</tr>
</tbody>
</table>

Fig. 3. The power control error statistics with a measurement error of standard deviation equal to 5 dB, i.e. \( \sigma_Y = 5 \) dB, with \( f_d = 20 \) Hz, and target \( E_b/N_0 = 5 \) dB.

IV. NUMERICAL RESULTS

In this section, we simulate the uplink performance for a single user under a flat Rayleigh fading channel. We assume the measurement errors are log-normal distributed random variables [13]. Other parameters used in the simulation are listed in Table I.

Through simulation, Figure 3 shows the probability density function of power control errors (PCE) subject to measurement errors with standard deviation of 5 dB. As shown in the figure, the PCE (expressed in dB domain) exhibits the similar characteristics of normal distribution. This result confirms our assumption that the PCE is lognormal distributed random variable even with the influence of measurement errors.

Figure 4 shows the impact of measurement errors for different Doppler frequencies on the probability of false power control command, \( P_{FC} \). One can note that the higher the Doppler frequency, the lower the \( P_{FC} \). This phenomenon can be explained as follows. Because the radio channel will change faster at a higher Doppler frequency, the power control mechanism usually cannot follow fast channel variations. Thus, in this situation a larger power control error will occur. Due to a larger gap between the target SIR and the actual SIR, it is less likely to change the power control command from “power up” to “power down” or vice versa. In other words, a larger PCE due to higher Doppler frequency can tolerate large measurement errors, thereby having a smaller \( P_{FC} \) as shown in the figure. In Figure 4, we also verify the accuracy of the analytical results of \( P_{FC} \) derived from (12) by comparing with simulation. As shown, simulation results of \( P_{FC} \) are close to the analytical results derived from (12).

Figure 5 shows the BER performance of Fig. 4 including the impact of \( P_{FC} \). Although in Fig. 4 it is demonstrated that a higher Doppler frequency leads to a smaller \( P_{FC} \), the BER performance at a higher Doppler frequency is still larger than that at a lower Doppler frequency as shown in Fig. 5. It is implied that even with a larger probability of false power control command, the closed-loop power control scheme under a slower fading channel still performs better than that under a fast fading channel.

Figure 6 shows the impact of very large measurement errors on the probability of false command \( P_{FC} \) in the closed-loop power control. One can see that as a measurement error increases up to 30 dB, the probability of false power control command tends to be saturated at 0.45. Note that in (16), the maximum of \( P_{FC} \) approaches to 0.5 for the infinite measurement error.

Figure 7 illustrates the BER performance of the closed-loop power control subject to very large measurement errors. As shown in the figure, the theoretical values according to formula (16) provide a reasonable bound on the BER performance of the closed-loop power control subject to measurement errors. Because \( P_{FC} \) will be bounded to be 0.5 even with large measurement errors, one can find that the
Fig. 5. The BER performance comparison of the closed-loop power control subject to measurement errors for different Doppler frequencies, where $f_d = 30$, 15, and 5 Hz, and target $E_b/N_0 = 5$ dB.

BER of the up-down closed-loop power control will also be bounded under a large measurement error. In the figure, with the target $E_b/N_0$ equal to 10 dB, the BER is limited to below 0.02 even with a measurement error with standard deviation of 30 dB. Thus, it is implied that the up/down power control scheme is robust to measurement errors.

Figure 8 compares the up/down closed-loop power control with the variable-step size closed-loop power control from the standpoint of the sensitivity of power control errors with respect to measurement errors. In our simulation, the variable-step size power control is a power control mechanism with higher resolution in power adaptation steps. As shown, unlike the variable-step power control has a strong correlation between measurement errors and power control errors, the impact of a larger measurement error on the up/down closed-loop power control is less sensitive.

V. Conclusions

In this paper we have presented a simple and accurate analytical expression for the probability of false command ($P_{FC}$) in the closed-loop power control (CLPC) subject to measurement errors. Moreover, we also propose a BER performance bound in terms of $P_{FC}$. It is found that the larger the Doppler frequency, the smaller the $P_{FC}$, while for the BER performance, a larger Doppler frequency results in a poor BER performance even with a smaller $P_{FC}$. Interestingly, we find that compared with the variable-step power control, the up/down CLPC is more robust to measurement errors due to the non-linear operation in the up/down power control scheme. The proposed analytical approach can be easily applied to analyze the performance of the closed-loop power control with different SIR measurement schemes.

Thus, an interesting future research extended from this work is to apply the proposed analytical framework to incorporate different SIR measurement schemes to evaluate the performance of the closed-loop power control in CDMA systems.

REFERENCES

Fig. 8. Comparison of the up/down closed-loop power control and the variable step closed-loop power control in terms of the correlation of power control errors and measurement errors for


