Effects of Multipath Fading on Delay-Locked Loops for Spread Spectrum Systems

Wern-Ho Sheen, Member, IEEE, and Gordon L. Stüber, Member, IEEE

Abstract—A renewal process approach is used to analyze noncoherent delay-locked loops (DLLs) for spread spectrum systems operating over frequency selective and frequency nonselective slow fading channels. The effects of multipath fading are evaluated in terms of the mean-time-to-lose-lock (MTLL) and the root-mean-square rms tracking error. For channels that have a specular component, the following results are observed: (i) The effect of multipath fading is more significant for tracking loops that have large loop signal-to-noise ratios (SNRs); (ii) A smaller early-late discriminator offset $\Delta$, which is bounded by $0 < \Delta < 1$, results a better tracking error performance. However, the tracking error is insensitive to $\Delta$ if there is a strong multipath fading effect and/or the tracking loops have small loop SNRs; (iii) A larger $\Delta (>0.5)$ may result in a higher MTLL, especially if there is a strong multipath effect. From (ii) and (iii), instead of using the popular choice of $\Delta = 0.5$, a larger $\Delta$ may be chosen for a DLL that works at low SNRs and/or strong multipath fading environments, where MTLL is an important consideration.

I. INTRODUCTION

Spread spectrum receivers require a synchronized version of the employed pseudo-noise (PN) spreading sequence so as to despread the received signal and allow recovery of the data sequence. Hence, a PN code synchronizer is an essential element in the design of spread spectrum receivers. Code synchronization is usually achieved in two steps: code acquisition followed by code tracking. During the code acquisition process the phase of the incoming PN sequence is determined to within a chip duration or less. Code tracking refers to the process of achieving and maintaining fine alignment of the chip boundaries of the incoming and locally generated PN sequences.

A typical PN code synchronizer for a direct sequence (DS) spread spectrum system is shown in Fig. 1. During code acquisition, the acquisition unit continually adjusts the phase of the local code until the incoming and local codes are aligned, so that the code phase error $\varepsilon(t)$ is within the permissible range $[\varepsilon_{\min}, \varepsilon_{\max}]$. The code phase error $\varepsilon(t) \triangleq [\tau_1(t) - \hat{\tau}_1(t)]/T_c$ is defined as the normalized phase difference between the incoming and local codes, where $\tau_1(t)$ and $\hat{\tau}_1(t)$ are the absolute phases of the incoming and local codes, respectively, and $T_c$ is the chip duration. For a code tracking loop, the permissible range $[\varepsilon_{\min}, \varepsilon_{\max}]$ is usually the range of the discriminator characteristic or S curve of the code tracking loop, i.e., the range for which the S curve is not zero. Whenever the code phase error is within this permissible range, there is a probability that the lock detector will declare that the synchronizer is in-lock and switch the synchronization to the code tracking unit to obtain fine alignment of the chip boundaries. This probability is a function of the code phase error and is described by the probability density function (pdf) $\pi(\varepsilon(t))$. It is also appropriate to refer to $\pi(\varepsilon(t))$ as the pdf of the initial code phase error for the code tracking process. The pdf $\pi(\varepsilon(t))$ depends on the particular acquisition unit. For a properly working acquisition unit, we usually have $\pi(0) \geq \pi(\varepsilon(t))$.

During the code tracking process the code phase error might exceed the permissible range $[\varepsilon_{\min}, \varepsilon_{\max}]$ because of the presence of channel dynamics and system noise, and cause the code synchronizer to be out-of-lock. In this case, the lock detector will trigger a reacquisition process and switch the PN code synchronizer back to the code acquisition unit. Moreover, there is still a probability, $P_{LD}(\varepsilon(t))$, that the lock detector will mistakenly trigger for a reacquisition even though the code phase error is within the permissible range $[\varepsilon_{\min}, \varepsilon_{\max}]$. It is assumed that $P_{LD}(\varepsilon_{\max}) = P_{LD}(\varepsilon_{\min}) = 1$, i.e., the boundaries are absorbing. This assumption is certainly reasonable for

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\[ |\varepsilon_{\text{max}}| \text{ and } |\varepsilon_{\text{min}}| \text{ in excess of } T_c, \text{ provided that the spreading sequences have low off-peak autocorrelations.}

From the above discussion, the PN code synchronizer follows a combined tracking/reacquisition process after initial code acquisition. In this paper, only code tracking process will be considered. The code tracking process can be treated separately from the code reacquisition process, if one is only interested in the stationary behavior of the code tracking loops [1]. In effect, the individual code tracking processes will be treated as being statistically identical.

Two configurations for code tracking loops have been considered extensively in the literature: delay-locked loops (DLLs) [2,3] and their time-shared modification tau-dither loops (TDLs) [4]. TDLs can cope with the gain imbalance problem of DLLs, but suffer from an approximate 3 dB loss in tracking errors. Both DLL and TDL can be applied in coherent and noncoherent modes. Coherent tracking loops do not have squaring losses and gain imbalances. However, noncoherent loops are more often employed in spread spectrum applications, because the spread energy-to-noise ratio is typically too low to obtain carrier recovery before code synchronization.

Linear and nonlinear approaches have been employed for analyzing code tracking loops operating on additive white Gaussian noise (AWGN) channels [1,5,6,7]. For AWGN channels, a linear analysis is adequate for sufficiently large loop signal-to-noise ratios (SNRs). However, for SNRs below a certain threshold, a nonlinear analysis is required [1]. For multipath fading channels, a nonlinear analysis is usually more appropriate, especially if there is a weak specular component and/or a significant delay spread. Unlike the case of a carrier phase-locked loop, the \( S \) curve or discriminator characteristic of the code tracking loop is, in effect, nonperiodic and of finite range, because of the phenomenon of boundary absorption. That is, whenever the code phase error reaches one of the boundaries \( \varepsilon_{\text{min}} \) and \( \varepsilon_{\text{max}} \), the tracking process will be terminated and the synchronization will be switched back to the code acquisition unit for a reacquisition. Hence, the renewal process approach (RPA) must be invoked to provide an exact nonlinear analysis [1,6,7]. The RPA was first proposed by Meyr [6,7] and has been applied by Polydoros and Weber [1] for analyzing the performance of noncoherent DLLs.

In the literature, most analyses (linear or nonlinear) of code tracking loops have been carried out for AWGN channels [1,5]. One exception is the work by Bogusch et al. [9] where, among other things, the effect of frequency-selective multipath fading on the performance of coherent and noncoherent code tracking loops has been evaluated by computer simulations. In this paper, the RPA is used to analyze the performance of noncoherent DLLs operating over slowly time-varying multipath fading channels. Both frequency-selective and -selective multipath fading channels are considered. The effects of multipath fading are evaluated in terms of the mean-time-to-lose-lock (MTLL) and root-mean-square (rms) tracking errors.

The remainder of the paper is organized as follows. Section II describes the system and channel model. In Section III, the stochastic differential equation which describes the behavior of the code tracking loop is first derived, and then the rms tracking error and MTLL are obtained by solving a Fokker-Plank equation. In Section IV, the effects of multipath fading on the performance of a noncoherent DLL are presented. Section V presents our numerical results and, finally, some concluding remarks are given in Section VI.

II. SYSTEM AND CHANNEL MODEL
For a discrete multipath fading channel, the complex low-pass impulse response can be expressed as [10]

\[
h(\tau; t) = \sum_n \left\{ A_n(t) + a_nR(t) + j a_{nI}(t) \right\} \times \delta(\tau - \tau_n(t))
\]

(1)

where \( A_n(t) \) is the specular component from the direct path, \( a_nR(t) \) and \( a_{nI}(t) \) are diffuse components, and \( \tau_n(t) \) is the associated time delay. The diffuse components are the gross effect of a large number of independent paths of delay \( \tau_n(t) \). Thus, by applying the Central Limit Theorem, \( a_nR(t) \) and \( a_{nI}(t) \) can be modeled as uncorrelated zero-mean Gaussian random processes with equal variance. The channel is assumed to be wide sense stationary with uncorrelated scattering; the latter implies that the channel impulse response in (1) is uncorrelated in the delay variable \( \tau \).

The impulse response in (1) can also be written in the polar form

\[
h(\tau; t) = \sum_n g_n(t) e^{j \tilde{\theta}_n(t)} \delta(\tau - \tau_n(t))
\]

(2)

where

\[
g_n(t) = \sqrt{[A_n(t) + a_nR(t)]^2 + a_{nI}(t)^2}
\]

(3)

and

\[
\tilde{\theta}_n(t) = \tan^{-1} \left( \frac{a_{nI}(t)}{A_n(t) + a_nR(t)} \right)
\]

(4)

are the attenuation and phase associated with the nth path, respectively. Since \( a_nR(t) \) and \( a_{nI}(t) \) are zero-mean, independent, and identically distributed Gaussian random variables, \( g_n(t) \) is Rician distributed with pdf\(^1\)

\[
p(g_n) = \frac{g_n}{\sigma_n^2} \exp \left( -\frac{g_n^2 + A_n^2}{2\sigma_n^2} \right) I_0 \left( \frac{g_n A_n}{\sigma_n^2} \right), \quad g_n \geq 0
\]

(5)

where \( \sigma_n^2 = \text{E}[a_n^2 R^2] = \text{E}[a_n^4] \), \( I_0(\bullet) \) is zeroth order modified Bessel function of the first kind, and \( \text{E}(\cdot) \) denotes the expectation operation. If \( A_n = 0 \), then the well known Rayleigh fading model is obtained with

\[
p(g_n) = \frac{g_n}{\sigma_n^2} \exp \left( -\frac{g_n^2}{2\sigma_n^2} \right), \quad g_n \geq 0
\]

(6)

and \( \tilde{\theta}_n \) uniformly distributed on \([0, 2\pi]\).

\(^1\)In the sequel, the time variable \( t \) associated with \( g_n(t) \), \( \tilde{\theta}_n(t) \), and \( \tau_n(t) \) is omitted for notational simplicity. Also, for slowly time-varying channels the time variable \( t \) can be omitted anyway.
For a DS/BPSK spread spectrum system, the transmitted signal \( x(t) \) is

\[
x(t) = \sqrt{2P}m(t)c(t) \cos(w_0t)
\]

where \( m(t) \) is the binary data sequence, \( c(t) \) is a PN sequence, \( P \) is the carrier power, and \( w_0 \) is the carrier frequency. In (7), the initial carrier phase is assumed to be zero. According to the channel model just given, the received signal \( r(t) \) is

\[
r(t) = \sum_n \sqrt{2P}g_n m(t - \tau_n)c(t - \tau_n) \cos(w_0t + \theta_n) + n(t)
\]

where \( n(t) \) is AWGN with two-sided power spectral density \( N_0/2 \) watts/Hz, and \( \theta_n = \theta_n - w_0\tau_n \). In this paper, only two-path channels will be considered, so that the received signal \( r(t) \) becomes

\[
r(t) = \sqrt{2P} \left\{ g_1 m(t - \tau_1)c(t - \tau_1) \cos(w_0t + \theta_1) \right. \\
+ g_2 m(t - \tau_2)c(t - \tau_2) \cos(w_0t + \theta_2) \right\} + n(t)
\]

where \( \tau_2 = \tau_2 - \tau_1 \) is the delay difference between the first path and the second path. Without loss of generality, the second path can be considered to be the main path. That is, \( \tau_1 \) is the time delay to be tracked by the code tracking loop. In the following, the noise \( n(t) \) will be written in the quadrature form

\[
n(t) = \sqrt{2} \left\{ n_c(t) \cos(w_0t + \theta_1) \\
- n_s(t) \cos(w_0t + \theta_1) \right\}
\]

where \( n_c(t) \) and \( n_s(t) \) are zero-mean, of equal variance, and mutually independent Gaussian random processes with the same power spectral density as \( n(t) \).

III. NONCOHERENT EARLY-LATE DELAY-LOCKED LOOPS

A noncoherent DLL is shown in Fig. 2. The received signal \( r(t) \) is first correlated with the locally generated early \( c(t - \tau_1 + \Delta T_c) \) and late \( c(t - \tau_1 - \Delta T_c) \) PN codes. The desired code phase error signal \( e(t) \) is then obtained by band-passing filtering, squaring, and differencing the correlator outputs. The squaring operations remove the effects of data modulation and carrier phase shift. The loop is closed by low-pass filtering the error signal, which is used to drive the Voltage Control Clock (VCC) and correct the code phase error of the local PN code generator. The parameter \( \Delta, 0 < \Delta < 1 \) is called the early-late discriminator offset. The general choice for the early-late discriminator offset is \( \Delta = 0.5 \). It will be shown, however, that \( \Delta > 0.5 \) may be a better choice for some particular working environments.

\[\text{Fig. 2 A noncoherent delayed-locked loop for direct sequence spread spectrum systems.}\]

A. Stochastic Differential Equation

From Fig. 2, the signals at the output of the band-pass filters (BPFs), neglecting the code self noise, are

\[
z_{\pm}(t) = \sqrt{2P} \left\{ g_1 m_f(t - \tau_1) R_c(\varepsilon \pm \Delta) \cos(w_0t + \theta_1) \\
+ \sqrt{2P} \left\{ g_2 m_f(t - \tau_1 - \tau_2) R_c(\varepsilon \pm \Delta + \frac{\tau_d}{T_c}) \cos(w_0t + \theta_2) \\
+ \sqrt{2} \left\{ w_c^\pm(t) \cos(w_0t + \theta_1) \\
- w_s^\pm(t) \sin(w_0t + \theta_1) \right\} \right\}
\]

where

\[
m_f(t) \triangleq m(t) \ast h_1(t)
\]

\[
R_c(\xi) \triangleq \frac{1}{N T_c} \int_0^{N T_c} c(t + \xi T_c)dt
\]

\[
w_{\pm}^c(t) = n_c(t)[c(t - \tau_1 \pm \Delta T_c) \ast h_1(t)
\]

\[
w_{\pm}^s(t) = n_s(t)[c(t - \tau_1 \pm \Delta T_c) \ast h_1(t)
\]

\( N \) is the period of the PN sequence, and \( h_1(t) \) is the impulse response of the equivalent low-pass filter. Note that \( R_c(\xi) \) is periodic with period \( N \). Also, for a large \( N \), \( R_c(\xi) \) can be approximated in the interval \([-\frac{N}{2}, \frac{N}{2}] \) by the ideal autocorrelation

\[
R_c(\xi) = \left\{ \begin{array}{ll}
1 - |\xi| & \text{if } |\xi| \leq 1 \\
0 & \text{otherwise}
\end{array} \right.
\]

Let \( z_{\pm}(t) \text{lp} \) denote the low-pass portions of the signals at the output of square-law devices. Then, by neglecting the modulation self-noise, the error signal \( e(t) \) is:

\[
e(t) \triangleq z_{\pm}^2(t) \text{lp} - z_{\pm}^2(t) \text{lp} = PM(0) \left\{ g_1^2 h_1(\varepsilon, 0) + g_2^2 h_1(\varepsilon, \frac{\tau_d}{T_c}) \right\}
\]

2In writing Eq. (11), it is assumed that the channel is slowly time-varying.

3This expression also assumes a rectangular chip shaping function.
\[ + 2g_1g_2 \frac{M(\tau_d)}{M(0)} h_2(\epsilon, \frac{\tau_d}{T_c}) \cos(\theta_1 - \theta_2) \bigg] \]
\[ + n_1(t) + n_2(\epsilon, t) + n_3(\epsilon, t) \]  \hspace{1cm} (17)

where

\[ M(\xi) = \left\langle E[m_f(t)m_f(t + \xi)] \right\rangle \]
\[ = \int_0^\infty S_m(f)|H(f)|^2e^{j2\pi f\xi} df \]  \hspace{1cm} (18)

\[ h_1(\epsilon, \xi) = R_2^\prime(\epsilon - \Delta + \xi) - R_2^\prime(\epsilon + \Delta + \xi) \]  \hspace{1cm} (19)

\[ h_2(\epsilon, \xi) = R_c(\epsilon - \Delta) R_c(\epsilon + \Delta + \xi) - R_c(\epsilon + \Delta) R_c(\epsilon + \Delta + \xi) \]  \hspace{1cm} (20)

\[ n_1(t) = w_c^-(t) + w_c^+(t) \]
\[ n_2(\epsilon, t) = 2\sqrt{P}g_1m_f(t - \tau_1) \left| R_c(\epsilon - \Delta) w_c^-(t) - R_c(\epsilon + \Delta) w_c^+(t) \right| \]  \hspace{1cm} (21)

\[ n_3(\epsilon, t) = 2\sqrt{P}g_2m_f(t - \tau_1 - \tau_d) \]
\[ \times \left[ R_c(\epsilon - \Delta + \frac{\tau_d}{T_c}) w_c^-(t) - R_c(\epsilon + \Delta + \frac{\tau_d}{T_c}) w_c^+(t) \right] \]  \hspace{1cm} (22)

with

\[ \dot{w}_c^\pm(t) = w_c^\pm(t) \cos(\theta_1 - \theta_2) - w_c^\pm(t) \sin(\theta_1 - \theta_2) \]  \hspace{1cm} (24)

In (18), \( \langle \cdot \rangle \) denotes time average, \( S_m(f) \) is the power spectral density of the data signal \( m(t) \), and \( H(f) \) is the transfer function of the BPF. Note that if \( \tau_d \) is much less than the bandwidth of the bandpass filter, then \( M(\tau_d) \approx M(0) \).

From Fig. 2, the error signal is passed through the loop filter and its output drives the VCC to correct the code phase error. The operation of the VCC is described by the equation

\[ \frac{\dot{\tau}_1(t)}{T_c} = k_L \int_0^t f(t') * e(t') dt' \]  \hspace{1cm} (25)

where \( k_L \) is the VCC gain, and \( f(t) \) is the impulse response of the loop filter. Combining (17) and (25) gives the following stochastic differential equation which describes the dynamic behavior of the tracking loop.

\[ \frac{d\epsilon(t)}{dt} = \beta_D - k_L \left\{ PM(0)[S_D(\epsilon) + S_I(\epsilon)] + n_T(\epsilon, t) \right\} * f(t) \]  \hspace{1cm} (26)

where

\[ \beta_D = \frac{1}{T_c} \frac{dr_1(t)}{dt} \]  \hspace{1cm} (27)

\[ S_D(\epsilon) = g_1^2 h_1(\epsilon, 0) \]  \hspace{1cm} (28)

\[ S_I(\epsilon) = g_2^2 h_1(\epsilon, \frac{\tau_d}{T_c}) \]
\[ + 2g_1g_2 \frac{M(\tau_d)}{M(0)} h_2(\epsilon, \frac{\tau_d}{T_c}) \cos(\theta_1 - \theta_2) \]  \hspace{1cm} (29)

\[ n_T(\epsilon, t) = n_1(t) + n_2(\epsilon, t) + n_3(\epsilon, t) \]  \hspace{1cm} (30)

In (26), \( \beta_D \) is the Doppler shift, \( S_D(\epsilon) \) is the desired discriminator characteristic, and \( S_I(\epsilon) \) is the interference that is caused by the effect of the second path. Fig. 3 shows the overall discriminator characteristic \( S(\epsilon) = S_D(\epsilon) + S_I(\epsilon) \) for the case of \( \Delta = 0.5, g_1^2 = 1, g_2^2 = 0.5, \tau_d = 0.5T_c \) and \( \theta_1 = \theta_2 \). Note that unlike the behavior of \( S_D(\epsilon) \), the linear region around \( \epsilon = 0 \) of the overall discriminator characteristic may not exist when there is a multipath effect. In this case, a nonlinear analysis is more appropriate for analyzing the performance of the tracking loop. Also, because of the phenomenon of boundary absorption, the range of overall discriminator characteristic is, in effect, non-periodic and finite. Hence, the RFA is employed in the sequel to obtain the exact nonlinear analysis.

### B. Nonlinear Analysis

In the nonlinear analysis, the stationary rms tracking error \( \sigma_{rms} \) and the MTLL, \( E[\tau'^2] \), are usually employed for evaluating the performance of a code-tracking loop [1,5,6]. The rms tracking error and the MTLL characterize a code-tracking loop in the same way that the rms phase error and cycle-slip rate characterize a carrier-tracking loop. Note that when the SNR is large and the Doppler shift is negligible, the MTLL is usually very large and the tracking error becomes the dominant performance criterion for a tracking loop. However, for a low SNR or a non-negligible Doppler shift or both, the MTLL becomes important when evaluating a tracking loop. It will be shown that these two quantities are closely related to the stationary pdf of the tracking error \( \epsilon \). In the following, only a first-order DLL will be considered\(^4\).

\(^4\)The method here can also be applied to analyze second and higher
Let \( p(\varepsilon|g) \) be the stationary pdf of the tracking error conditioned on the channel impulse response \( g \). According to RPA [1,6], \( p(\varepsilon|g) \) satisfies the following Fokker-Plank equation with the boundary conditions \( p(\varepsilon_{\text{min}}|g) = p(\varepsilon_{\text{max}}|g) = 0 \):

\[
\frac{\partial}{\partial \varepsilon} \left( k_1(\varepsilon)p(\varepsilon|g) \right) - \frac{1}{2} \frac{\partial^2}{\partial \varepsilon^2} \left[ k_2(\varepsilon)p(\varepsilon|g) \right] = \pi(\varepsilon) \frac{E[\tau^r|g]}{E[\tau^r]} \]

where \( P_0(\varepsilon) = 1 - P_{LD}(\varepsilon) \), \( E[\tau^r|g] \) is the conditional MTLL,

\[
k_2(\varepsilon) = k_L^2 \int_{-\infty}^{\infty} R_{n\tau}(\varepsilon, \xi) d\xi \quad (32)
\]

\[
k_1(\varepsilon) = \beta_D - k_L^2 \text{PM}(0)[S(\varepsilon) + S_I(\varepsilon)] + \frac{d}{4} k_2(\varepsilon) \quad (33)
\]

and \( R_{n\tau}(\varepsilon, \xi) = E[n_{\tau}(\varepsilon, t) n_{\tau}(\varepsilon, t + \xi)] \) is the autocorrelation function of the total noise \( n_{\tau}(\varepsilon, t) \) in (30). In (31), \( \pi(\varepsilon) \) and \( P_0(\varepsilon) \) may vary with different channel impulse responses \( g \), but for the sake of simplicity they are assumed to be fixed functions of \( \varepsilon \). Following the procedure in Lindsey [8], the conditional pdf \( p(\varepsilon|g) \) and MTLL \( E[\tau^r|g] \) can be obtained as follows:

\[
E[\tau^r|g] = \int_{-\infty}^{\infty} Q(\varepsilon|g) d\varepsilon
\quad (34)
\]

and

\[
p(\varepsilon|g) = \frac{Q(\varepsilon|g)}{E[\tau^r|g]} \quad , \quad \varepsilon_{\text{min}} \leq \varepsilon \leq \varepsilon_{\text{max}}
\quad (35)
\]

where

\[
Q(\varepsilon|g) = \frac{2 P_0(\varepsilon) e^{-u_0(\varepsilon)}}{k_2(\varepsilon)} \times \int_{\varepsilon_{\text{min}}}^{\varepsilon} \frac{u_0(x)}{d_0 - \frac{\pi(x)}{x}} dx \quad (36)
\]

with

\[
u_0(\varepsilon) = -2 \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} k_1(x) k_2(x) dx \quad (37)
\]

\[
d_0 = \int_{\varepsilon_{\text{max}}}^{\varepsilon_{\text{min}}} \frac{\tau^r}{\tau_{\text{min}}} \pi(x) dx \quad \pi(x) = \int_{\varepsilon_{\text{min}}}^{\varepsilon} \pi(x) dx \quad (39)
\]

Using the conditional pdf in (35), the conditional rms tracking error is:

\[
\sigma_{\varepsilon|g}^{\text{rms}} = \sqrt{\int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} \varepsilon^2 p(\varepsilon|g) d\varepsilon} \quad (40)
\]

Since the channel is slowly time-varying, the rms tracking error and the MTLL can be obtained by averaging (40) and (34), respectively, over the ensemble of channel impulse responses. That is,

\[
\sigma_{\varepsilon|g}^{\text{rms}} = \int_{g} \sigma_{\varepsilon|g}^{\text{rms}} p(g) d\varepsilon \quad (41)
\]

and

\[
E[\tau^r] = \int_{g} E[\tau^r|g] p(g) d\varepsilon \quad (42)
\]

where \( p(g) \) is the joint pdf of the channel impulse response. In general, closed form expressions for \( \sigma_{\varepsilon|g}^{\text{rms}} \) and \( E[\tau^r] \) do not exist and numerical methods must be employed to evaluate these quantities.

**IV. Effects of Multipath Fading**

In this section, the effects of a specular multipath fading channel on the performance of a DLL are determined. This type of channel may occur in an indoor or outdoor wireless environment where a direct path can be obtained by the appropriate deployment of base station antennas. For this type of environment the two-path channel model becomes

\[
h(\tau) = \sqrt{2P \left( \delta(\tau - \tau_1)e^{j\theta_1} + g_2e^{j\theta_2}\delta(\tau - \tau_1 - \tau_d) \right)} \quad (43)
\]

where \( \theta_1 \) is a constant phase shift, and \( g_2 \) and \( \theta_2 \) are Rayleigh- and uniform-distributed random variables, respectively. When \( \tau_d = 0 \), the channel becomes the familiar frequency non-selective Rician fading model.

In order to present our results, the following important system parameters are needed: the power ratio of the main path to the second path, \( R \), the bit SNR (SNR in data bandwidth), \( \gamma_d \), the loop SNR for \( \Delta = \frac{1}{2} \), \( \gamma_{L_0} \), and the ratio \( \zeta_0 = \gamma_{L_0}/\gamma_d \). The parameters \( R \), \( \gamma_d \), and \( \gamma_{L_0} \) are defined as follows:

\[
R \triangleq \frac{1}{E[g_2]} \quad (44)
\]

\[
\gamma_d \triangleq \frac{P T_b}{N_0} \quad (45)
\]

and

\[
\gamma_{L_0} \triangleq \frac{P}{N_0 B_L \frac{\Delta}{2}} \quad (46)
\]

where \( T_b \) is duration of an information bit, and \( B_L \) is the closed-loop bandwidth for the case when \( g_2 = 0 \). That is,

\[
B_L = \int_{-\infty}^{\infty} |H(f)|^2 df \quad (47)
\]

where \( H(s) \) is the close loop transfer function defined as

\[
H(s) \triangleq \frac{L(\tau(t))}{L(\tau_1(t))} = \frac{k_L M(0) PS_D(0)}{s + k_L M(0) PS_D(0)} \quad (48)
\]
with $\mathcal{L}\{\cdot\}$ denoting the operation of Laplace transform, and $S_D'(\varepsilon) = \frac{dS_D(\varepsilon)}{d\varepsilon}$. Note that $H(s)$ depends on $\Delta$ through $S_D'(0)$. It can be shown that

$$B_L|_{\Delta = \frac{1}{2}} = \frac{k_L M(0) P}{2}.$$  \hspace{1cm} (49)

From (45) and (46), the ratio $\zeta_0$ is

$$\zeta_0 = \frac{\gamma L_0}{\gamma_d} = \frac{R_b}{B_L|_{\Delta = \frac{1}{2}}}.$$  \hspace{1cm} (50)

where $R_b = 1/T_b$ is the information rate. From these definitions and from (32), along with the expression for $R_{nP}(\varepsilon, \xi)$ that is derived in Appendix A, we have

$$k_2(\varepsilon) = \frac{1}{M(0)\gamma L_0 T_i} \left\{ 8c_1[1 - q^2(\Delta)] + 8c_2\gamma_d \left[ h_3(\varepsilon, 0) - q(\Delta) h_4(\varepsilon, 0) \right] + g_2 h_3(\varepsilon, \frac{\tau_d}{T_i}) - q(\Delta) h_4(\varepsilon, \frac{\tau_d}{T_i}) + 2g_2 h_5(\varepsilon, \frac{\tau_d}{T_i}) - q(\Delta) h_6(\varepsilon, \frac{\tau_d}{T_i}) \times \cos(\theta_1 - \theta_2) \right\}$$  \hspace{1cm} (51)

and

$$k_1(\varepsilon) = \beta_D - \frac{2\gamma_d}{T_t\gamma L_a} [S(\varepsilon) + S_I(\varepsilon)] + \frac{1}{4} \frac{d}{d\varepsilon} k_2(\varepsilon).$$  \hspace{1cm} (52)

where

$$c_1 \triangleq \frac{T_b}{M(0)} \int_{-\infty}^{\infty} |H_i(f)|^4 df$$  \hspace{1cm} (53)

$$c_2 \triangleq \frac{1}{M(0)} \int_{-\infty}^{\infty} S_{\alpha}(f) H_i(f)^4 df.$$  \hspace{1cm} (54)

In (51), $q(\Delta)$ and $h_i$ for $i = 3, 4, 5, 6$ are defined in (A7), (A8), (A9), (A10), and (A11), respectively. After obtaining $k_1(\varepsilon)$ and $k_2(\varepsilon)$, the $rms$ tracking error and the MTLL can be calculated as previously described.

V. NUMERICAL RESULTS

The examples that follow assume a binary NRZ data sequence, and assume that an ideal BPF is used with cutoff frequency of $R_b$. With these assumptions, it can be shown that $M(0) = 0.902$, $c_1 = 2/0.902$, and $c_2 = 1$. Since the channels are slowly time-varying, $\beta_D$ is assumed to be zero. Finally, all results are obtained under the assumption that $\pi(\varepsilon) = \delta(\varepsilon)$ and

$$P_D(\varepsilon) = \begin{cases} 1 - \varepsilon^2 & \text{if } \varepsilon \leq 1 \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (55)

which implies that we have a very accurate acquisition unit, and boundary absorption occurs at $|\varepsilon| = 1$.

A. Two-Path Examples

The two-path model results when $\tau_d \neq 0$. Figs. 4 and 5 show the effects of multipath fading on the $rms$ tracking error for the cases $\zeta_0 = 10$ and $\zeta_0 = 100$, respectively. It is evident that the degradation due to multipath is more significant when the tracking loop has a higher loop SNR, $\gamma L_0 = \gamma_d \zeta_0$. Unfortunately, this is the range of loop SNRs of practical interest. This phenomenon can be attributed to the fact that, for large loop SNRs, the interference effect is dominated by the term $S_I(\varepsilon)$ as defined in (29). Figs. 4 and 5 also show that for $\zeta_0 = 10$ and $\gamma_d \geq 10$ dB, or for $\zeta_0 = 100$ and $\gamma_d \geq 5$ dB, the degradation due to multipath is present even for $R$ as large as $15$ dB.
Fig. 6 Effects of multipath fading on the MTLL performance with various delay spacings ($\Delta = 0.5, \zeta_0 = 100$).

Fig. 7 Effects of multipath fading on the tracking error performance with various early-late discriminator offsets ($\tau_d = 0.5, \zeta_0 = 100$).

Fig. 8 Effects of multipath fading on the MTLL performance with various early-late discriminator offsets ($\tau_d = 0.5, \zeta_0 = 100$).

Fig. 6 shows the effects of multipath fading on the MTLL. In our results, the normalized MTLL is defined as $B_{\mu}[\Delta] = \frac{1}{\sigma} \times \mathbb{E}[r^2]$. It is interesting to note that multipath fading increases the MTLL. However, this result has to be interpreted with some caution, because the DLL experiences a large averaged received power with multipath fading as compared to the case when there is no multipath fading. Fig. 6 also shows the phenomenon that the multipath effects are more significant for larger loop SNRs, $\gamma_L$.

Figs. 7 and 8 show the effects of early-late discriminator offset on the performance of the DLL. For low $\gamma_L$, the performance of the DLL is insensitive to the value of $\Delta$. However, for higher $\gamma_L$, the impact of multipath on the choice of $\Delta$ fading are more complicated in the following sense. A smaller $\Delta$ results in a better tracking error performance and this is also true for low SNR cases. However, for a larger delay spread the tracking error is insensitive with increasing early-late discriminator offset, $\Delta$. On the other hand, the MTLL may be improved when a larger $\Delta$ (> 0.5) is employed, and the improvement is more significant for larger delay spreads. Therefore, if the MTLL becomes a critical parameter in a particular application (for example with channels having a large delay spread or for tracking loops that have small loop SNRs or both), a larger $\Delta$ (> 0.5) may be chosen to increase the MTLL. Similar results have been obtained for different delay spacings $\tau_d$ and for the case when $\zeta_0 = 10$.

B. Single-Path Examples

For $\tau = 0$ the channel is affected by frequency nonselective Rician fading. In this case, $R$ is the Rice factor, defined as the ratio of the power in the specular component to the power in the diffuse component, i.e., $R = 1/2\sigma^2$ with $\sigma^2$ defined in (5). Figs. 9 and 10 show the effects of Rician fading on the $\text{rms}$ tracking error. From these figures, we observe again that the effects of Rician fading are more significant for larger loop SNRs $\gamma_L$ and that the $\text{rms}$ tracking error is insensitive to the value of the early-late discriminator offset $\Delta$ for low SNR and/or channels with a small Rice factor.

Figs. 11 and 12 show the effects of Rician fading on the MTLL. For larger $\Delta$ (> 0.5) a higher MTLL may be obtained, especially if there is a small Rice factor. Therefore,
the same type of conclusion is reached as in two-path case: for low SNRs and/or channels with a weak specular component, a larger $\Delta (> 0.5)$ may be chosen to obtain a higher MTLL, because the $rms$ tracking error is relatively insensitive to the value of $\Delta$. It is worthy to note that, unlike the $rms$ tracking error, Rician fading might have a positive effect on the MTLL for larger $\zeta_0$. This is because the conditional MTLL is highly sensitive to changes in the channel attenuation factor for large $\zeta_0$. After averaging over all possible channel impulse responses, the averaged MTLL that is obtained might be larger than the MTLL that is obtained without the effects of fading. This positive effect might be important for the cases where the value of MTLL becomes a consideration.

VI. CONCLUDING REMARKS
In this paper, the renewal process approach is applied to analyze noncoherent DLLs for spread spectrum systems operating over frequency-selective and nonselective slowly fading channels. The effects of multipath fading are eval-
uated in terms of the MTTL and the \(\text{rms}\) tracking error. The results that were obtained are summarized as follows:

1. Multipath fading has a more significant effect on tracking loops that have a large loop SNR, because in this case the interference is dominated by the interfering discriminator characteristic \(S_f(\xi)\) rather than the additive noise. In some cases, the effects are observed to be large even when the ratio of the power between the main path and the interfering path is as large as 15 dB.

2. A smaller early-late discriminator offset \(\Delta\), which is bounded by \(0 < \Delta < 1\), results a better tracking error performance. However, the tracking error is insensitive to \(\Delta\) if there is a strong multipath fading effect and/or the tracking loops have small loop SNRs. Note that in practical applications, the lowest value of \(\Delta\) may be limited by the clock rate that is required to implement the fractional shift \(\Delta T_c\) of the chip duration.

3. In practice, the popular choice for \(\Delta\) is 0.5. However, our results show that a larger \(\Delta\) (> 0.5) may result in a higher MTTL, especially if there is a strong multipath effect. Therefore, from (ii) and (iii), one may choose \(\Delta > 0.5\) for a DLL which works at low SNR and/or strong multipath fading environments, where MTLL is an important consideration.

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APPENDIX

DERIVATION OF \(R_{n_p}(\xi, \xi)\)

In this Appendix, the autocorrelation \(R_{n_p}(\xi, \xi)\) of the total noise \(n_p(\xi, t)\) is derived. From (30), the total noise is

\[
n_p(\xi, t) = n_1(t) + n_2(\xi, t) + n_3(\xi, t)
\]

where \(n_1(t)\), \(n_2(t)\), and \(n_3(\xi, t)\) are defined in (21), (22), and (23), respectively. Observe that \(n_1(t)\) is uncorrelated with \(n_2(\xi, t)\) and \(n_3(\xi, t)\). Hence,

\[
R_{n_p}(\xi, \xi) = R_{n_1}(\xi) + R_{n_2}(\xi, \xi) + R_{n_2}(\xi, \xi) + R_{n_3(\xi, \xi)} + R_{n_3(\xi, \xi)}
\]

with \(R_{n_3(\xi, \xi)} = R_{n_3(\xi, -\xi)}\). After some algebra, it can be shown that

\[
R_{n_1}(\xi) = 2N_o[1 - q^2(\Delta)]
\]

\[
\times \left[ \int_{-\infty}^{\infty} |H_1(f)|^2 e^{i2\pi \xi f} df \right]^2
\]

\[
R_{n_2}(\xi, \xi) = 2P g_2^2 M(\xi) N_o \left\{ h_3(\xi, 0) - q(\Delta) h_4(\xi, 0) \right\}
\]

\[
\times \int_{-\infty}^{\infty} |H_1(f)|^2 e^{i2\pi \xi f} df
\]

\[
R_{n_3(\xi, \xi)} = 2P g_1 g_2 M(\xi + \frac{\tau_d}{T_c}) N_o
\]

\[
\times \left\{ h_3(\xi, \frac{\tau_d}{T_c}) - q(\Delta) h_4(\xi, \frac{\tau_d}{T_c}) \right\}
\]

\[
\times \cos(\theta_1 - \theta_2) \int_{-\infty}^{\infty} |H_1(f)|^2 e^{i2\pi \xi f} df
\]

where

\[
q(\Delta) = \begin{cases} 
1 - 2\Delta & \text{if } 0 \leq \Delta \leq 1/2 \\
0 & \text{otherwise} 
\end{cases}
\]

\[
h_3(\xi, \xi) = R_2^2(\xi - \Delta + \xi) + R_2^2(\xi + \Delta + \xi)
\]

\[
h_3(\xi, \xi) = 2R_2(\xi - \Delta + \xi)R_2(\xi + \Delta + \xi)
\]

\[
h_3(\xi, \xi) = R_2(\xi - \Delta)R_2(\xi + \Delta)
\]

\[
h_3(\xi, \xi) = R_2(\xi - \Delta)R_2(\xi + \Delta)
\]

Combining the above results gives the autocorrelation

\[
R_{n_p}(\xi, \xi) = 2N_o^2 [1 - q^2(\Delta)] \left[ \int_{-\infty}^{\infty} |H_1(f)|^2 e^{i2\pi \xi f} df \right]^2
\]

\[
+ 2P N_o \int_{-\infty}^{\infty} |H_1(f)|^2 e^{i2\pi \xi f} df
\]

\[
\times \left\{ g_1^2 M(\xi)[h_3(\xi, 0) - q(\Delta) h_4(\xi, 0)]
\]

\[
+ g_2^2 M(\xi)[h_3(\xi, \frac{\tau_d}{T_c}) - q(\Delta) h_4(\xi, \frac{\tau_d}{T_c})]
\]

\[
+ 2g_1 g_2 M(\xi + \frac{\tau_d}{T_c}) + M(\xi - \frac{\tau_d}{T_c})
\]

\[
\times [h_3(\xi, \frac{\tau_d}{T_c}) - q(\Delta) h_4(\xi, \frac{\tau_d}{T_c})]
\]

\[
\times \cos(\theta_1 - \theta_2)
\]

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\[\text{Channels that exhibit a strong multipath effect have a small Rice factor and/or a significant delay spread.}\]


Wern-Ho Sheen (S’89-M’91) received the B.S. degree from the National Taiwan Institute of Technology, Taipei, Republic of China in 1982, the M.S. degree from the National Chiao Tung University, Hsinchu, Republic of China in 1984, and the Ph.D. degree from the Georgia Institute of Technology, Atlanta, USA in 1991, all in electrical engineering.

From 1984 to 1985 he was with Telecommunication Laboratories, Republic of China, where he was mainly involved in the projects of personal communications and basic rate ISDN. Since 1993 he has been an Associate Professor in the Department of Electrical Engineering at National Chung Cheng University, Republic of China. His research interests include cellular mobile and personal radio systems, and spread spectrum communications.

Gordon Stüber (S’61-M’88) received the B.A.Sc. and Ph.D. degrees in electrical engineering from the University of Waterloo, Waterloo, Ontario, Canada, in 1982 and 1986, respectively.

In 1986 he joined the School of Electrical and Computer Engineering, Georgia Institute of Technology, where he is currently an Associate Professor. His current research interests are in personal and mobile radio systems, spread-spectrum communications, adaptive equalization, and error control coding.

Dr. Stüber is an active member of the IEEE Communication and Vehicular Technology Societies, and is an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS.